



Complex Adaptive Space

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Preface

Why Complex Adaptive Space?

Most geometries begin by fixing a space and then studying what can exist inside it.

Complex Adaptive Space (CAS) inverts this order.

CAS begins with **motion, comparison, and context**, and asks a more primitive question:

What does it mean for something to remain the “same” when the rules themselves can change?

Classical geometry assumes that distance, angle, and shape are globally meaningful. Even when these assumptions are relaxed—as in differential geometry or topology—the underlying space remains structurally uniform. CAS is motivated by situations where this uniformity fails in an essential way: when **transport between contexts alters structure**, when **loops do not close**, and when **branching is unavoidable rather than pathological**.

Such situations already arise across mathematics and physics: - analytic continuation of complex functions

- multivalued phases and branch cuts
- holonomy and curvature
- context-dependent observables
- systems whose effective “laws” evolve with parameters

CAS does not attempt to replace these theories. Instead, it provides a **minimal geometric language** in which all of these phenomena are native rather than exceptional.

What CAS Is (and Is Not)

CAS is **not** a metric space, not a coordinate system, and not a deformation of Euclidean geometry.

CAS is a **transport-first geometry**.

Its primitives are: - **contexts**, where local structure exists, - **transports**, which move structure between contexts, - **composition**, which records path dependence, - **coherence**, which distinguishes meaningful closure from obstruction, - and **branching**, treated as intrinsic geometric data rather than an error.

In CAS: - Geometry is defined by **how structure changes**, not by what remains fixed. - Curvature appears as **holonomy**, not as a derived tensor. - Shapes are **not objects**, but **persistent relational patterns under transport**.

This perspective aligns CAS with relational and process-oriented views of mathematics, while remaining formally precise.

How to Read This Work

This work consists of two tightly related components:

1. Minimal Axioms of Complex Adaptive Space

A compact formal specification of what must exist in order to speak coherently about adaptive geometry: contexts, transports, holonomy, and branching.

2. Circles, Squares, and Triangles in CAS

Conceptual probes that translate classical geometric intuition into CAS language, showing how familiar figures re-emerge as transport-invariant patterns rather than rigid forms.

The axioms are intentionally **minimal and abstract**. They are not meant to teach CAS by themselves. Instead, they act as a **constraint system**: a guarantee that the intuitive constructions in the shape document are mathematically well-posed.

Readers unfamiliar with groupoids, holonomy, or branching are encouraged to **read the shapes document first**, treating circles, squares, and triangles as *diagnostic tools* rather than definitions.

A Simple Guiding Intuition

If a single intuition is needed, it is this:

CAS studies what survives motion when motion itself reshapes the rules.

A circle in CAS is not “a set of points at fixed distance,”
but a **radial relation that remains meaningful across contexts**.

A square is not “four equal sides,”
but a **symmetry cycle that closes coherently under transport**.

A triangle is not “three straight edges,”
but the **smallest loop capable of detecting curvature**.

In this sense, shapes in CAS behave more like *experiments* than objects: they reveal properties of the space through their failure—or success—to persist.

What Problems CAS Is Meant to Address

CAS is designed to clarify situations where classical geometry becomes strained or verbose: - When curvature is better understood as **failure to return**, not deviation from flatness. - When multi-valuedness is unavoidable and must be tracked structurally. - When global constants (length, angle, scale) lose invariant meaning. - When geometry must adapt to complex or evolving parameters.

Rather than forcing these phenomena into fixed frameworks, CAS treats them as **first-class geometric features**.

Scope and Future Directions

This work deliberately stops short of applications. That restraint is intentional.

CAS is presented here at the level of **foundations**, not models. Possible extensions—connections, costs, invariants, dynamics—are sketched only to show that the framework is fertile, not to constrain its use.

Future developments may include: - concrete CAS models on Riemann surfaces or discrete groupoids, - connections to quantum phases and topological systems, - adaptive notions of curvature and differentiation, - computational or categorical implementations.

These directions are left open so that CAS can remain a **general geometric language**, rather than a theory tied to a single domain.

Final Orientation

CAS asks the reader to temporarily let go of familiar geometric anchors and adopt a different stance:

Geometry is not what you draw.
Geometry is what endures.

The documents that follow formalize this stance with as little structure as possible—no more, and no less than is required for geometry to remain meaningful when context itself becomes part of the space.

Complex Adaptive Space — Minimal Axioms (Minimal Draft)

This document proposes a *minimal* axiomatic package for **Complex Adaptive Space (CAS)**: a space whose geometric meaning is induced by **transport between contexts** parameterised by (typically) complex periodicity.

The goal is to specify only what is needed to talk about: - contexts (where objects live), - transports (how objects travel), - composition/path dependence (holonomy), - and a notion of coherence (what “well-defined” means).

0. Primitives

A CAS consists of the following primitive data:

1. A set (or class) of **contexts** Ob .
2. For each ordered pair (A, B) of contexts, a set of **transports** $\text{Hom}(A, B)$.
3. A partial binary operation **composition**

$$\circ : \text{Hom}(B, C) \times \text{Hom}(A, B) \rightarrow \text{Hom}(A, C)$$

whenever the codomain/domain match.

4. For each context A , an **identity transport** $\text{id}_A \in \text{Hom}(A, A)$.

Optionally (often useful): 5. A **context parameter map** $\pi : \text{Ob} \rightarrow P$ where $P \subseteq \mathbb{C}$ (e.g., periodicities).

1. Groupoid Axiom (Local Invertibility)

Axiom G (Groupoid): The contexts and transports form a **groupoid**:

- **Associativity:** If $h \circ g$ and $(k \circ h) \circ g$ are defined then

$$k \circ (h \circ g) = (k \circ h) \circ g.$$

- **Identities:** For any $g \in \text{Hom}(A, B)$,

$$g \circ \text{id}_A = g, \quad \text{id}_B \circ g = g.$$

- **Inverses (local):** For each $g \in \text{Hom}(A, B)$ there exists $g^{-1} \in \text{Hom}(B, A)$ such that

$$g^{-1} \circ g = \text{id}_A, \quad g \circ g^{-1} = \text{id}_B.$$

Interpretation: transports are “reversible when they exist,” but do **not** require global closure under arbitrary operations outside admissible domains.

2. Complex Adaptivity Axiom (Context-Dependent Law)

Axiom A (Adaptivity): There is no global identification of objects across contexts; comparisons are mediated by transports.

Formally, each context A supports a local structure $\mathcal{S}(A)$ (numbers, fields, metrics, operators, etc.). A transport $g \in \text{Hom}(A, B)$ induces a (possibly partial) **pushforward/pullback** map between these local structures:

$$\mathcal{S}(g) : \mathcal{S}(A) \rightsquigarrow \mathcal{S}(B)$$

subject to functoriality where defined:

$$\mathcal{S}(\text{id}_A) = \text{id}_{\mathcal{S}(A)}, \quad \mathcal{S}(h \circ g) = \mathcal{S}(h) \circ \mathcal{S}(g).$$

Interpretation: structure “travels.” A CAS is a geometry where the *rules* can change with the context parameter (often complex).

3. Coherence Axiom (Holonomy as Data)

Closed paths need not be trivial.

Let Ω_A be the set of **loops** at A :

$$\Omega_A := \text{Hom}(A, A).$$

Axiom C (Coherence/Holonomy): There is a designated subset $\text{Coh}(A) \subseteq \Omega_A$ of **coherent loops** (or equivalently a predicate “coherent”) such that:

- $\text{id}_A \in \text{Coh}(A)$.
- If $\gamma, \delta \in \text{Coh}(A)$ then $\gamma \circ \delta \in \text{Coh}(A)$.
- If $g \in \text{Hom}(A, B)$ and $\gamma \in \text{Coh}(A)$, then

$$g \circ \gamma \circ g^{-1} \in \text{Coh}(B).$$

Define the **holonomy class** (obstruction) as

$$\text{Hol}(A) := \Omega_A / \text{Coh}(A).$$

Interpretation: path dependence is allowed. “Failure to return” is not an error; it is geometric information (curvature in context-space).

4. Complex Branching Axiom (Multi-valued Transport)

Complex parameters typically introduce branching (e.g., complex powers/logs).

Axiom B (Branching): For each context A and each transportable quantity $x \in \mathcal{S}(A)$, the transported value $\mathcal{S}(g)(x)$ may be **multi-valued**, but must be locally single-valued along any *chosen branch*.

Equivalently, CAS includes: - a **branch selection** rule along admissible paths, or - a refinement of contexts into **sheeted contexts** $\tilde{A}$ that make transports single-valued.

Interpretation: branch cuts are *structural obstructions* and belong to the geometry.

5. Isotropic Scaling Structure (Optional, but canonical)

A common CAS carries a distinguished **isotropic invariant** $c \in \mathbb{C} \setminus \{0\}$ and a base-growth field $f : \text{Ob} \rightarrow \mathbb{C} \setminus \{0\}$ such that for each context A with parameter $p = \pi(A)$,

$$\boxed{f(A) = c^{1/p}} \quad (\text{with branch management})$$

Minimal coherence requirement for this structure: - For any coherent loop $\gamma \in \text{Coh}(A)$, the induced action on $f(A)$ is trivial (preserves f within the chosen branch).

Interpretation: isotropy fixes a global invariant c , while local growth f adapts to complex periodicity p .

6. Minimal Notion of “Geometry” in CAS

A CAS becomes a geometric object once you define any one of the following (all compatible with the axioms above):

1. **Cost/length of transports:** $\ell : \text{Hom}(A, B) \rightarrow [0, \infty]$ (or $\mathbb{C}$ -valued), additive under composition.
2. **Connection-like rule:** a way to compare infinitesimal transports (an “adaptive derivative”).
3. **Invariant observables:** $I(A) \subseteq \mathcal{S}(A)$ preserved (up to coherence) under transport.

These are *extra structure*, not required for the minimal axioms.

7. Immediate Consequences

1. **Euclidean space appears as a collapse:** If $\text{Coh}(A) = \Omega_A$ for all A (all loops coherent) and branching is trivial, then transport is path-independent and CAS collapses to a fixed-structure geometry.
 2. **Complex periodicity forces branching:** If $\pi(A) \in \mathbb{C}$ and $f = c^{1/p}$ is used, then multi-valuedness is generic unless contexts are sheeted.
 3. **Curvature = holonomy:** Nontrivial $\text{Hol}(A)$ encodes transport curvature in context-space.
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8. Minimal Example Schema

- Contexts: $\text{Ob} = \{A_p : p \in P \subseteq \mathbb{C}\}$.
 - Transports: generated by admissible changes $p \rightarrow q$ with composition along paths.
 - Structure: $\mathcal{S}(A_p) = \mathbb{C}^\times$ and $f(A_p) = c^{1/p}$.
 - Coherence: loops that do not cross branch cuts (or whose net winding yields integer multiples consistent with the chosen sheet).
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9. Notes on Minimality

- Axioms G + A are the core of “adaptive space.”
- Axiom C is the minimal addition that makes path dependence *structural*.
- Axiom B is the minimal addition that makes the framework genuinely *complex-adaptive* (complex periodicity without ignoring branch phenomena).
- The isotropic law is optional structure, but it is a canonical motivating example.

Circles, Squares, and Triangles in Complex Adaptive Space (CAS)

Introduction

In **Complex Adaptive Space (CAS)**, geometry is not defined by fixed coordinates or immutable metrics. Instead, it is defined by **contexts**, **transports**, and **coherence**. Shapes do not exist as static objects; they exist as **relations that persist (or fail to persist) under transport between contexts**.

This document describes what **circles**, **squares**, and **triangles** mean within CAS, following the minimal axioms of groupoid structure, adaptivity, holonomy, and branching.

1. Circles in CAS — Transport-Invariant Radial Relations

Classical intuition

A circle is the set of points at a fixed distance from a center.

CAS definition

A **CAS circle** is a family of locally defined loci across contexts such that:

- Each context A possesses its own notion of distance or norm d_A
- A circle in context A is defined by a constraint of the form:

$$d_A(x, c_A) = r_A$$

- Under a transport $g : A \rightarrow B$, this constraint is pushed forward to:

$$d_B(x, c_B) = r_B$$

where r_B is determined by transport, possibly complex-valued or branch-dependent

Essential features

- The **radius is not fixed globally**; it adapts to the context
- Transport around a closed loop may:
 - change the phase of the radius
 - move the circle onto a different branch
 - introduce multi-sheeted structure
- Circle-ness is preserved **up to coherence**, not exact equality

Interpretation

A CAS circle is not a rigid curve but a **transport-stable radial relationship**.

2. Squares in CAS — Discrete Symmetry Cycles

Classical intuition

A square is a polygon with four equal sides and four right angles.

CAS definition

A **CAS square** is defined by:

- A closed cycle of four transport segments
- Equality relations between edge-classes under transport
- Coherent closure after four compositions

Angles, straightness, and even planarity are **not invariant**.

Structural description

Let

$$A_1 \xrightarrow{g_1} A_2 \xrightarrow{g_2} A_3 \xrightarrow{g_3} A_4 \xrightarrow{g_4} A_1$$

be a closed transport loop.

The square is the **equivalence class of this cycle** under coherence.

Consequences

- In different contexts, the square may appear:
 - skewed
 - curved
 - rhombic
 - partially collapsed
- After a full loop in context-space, the square may return:
 - rotated
 - mirrored
 - branch-shifted

Interpretation

A CAS square is a **symmetry pattern encoded in transport**, not a rigid polygon.

3. Triangles in CAS — Minimal Holonomy Objects

Classical intuition

A triangle consists of three points joined by straight edges.

CAS definition

A **CAS triangle** is the smallest nontrivial closed transport loop:

$$A \xrightarrow{g_{AB}} B \xrightarrow{g_{BC}} C \xrightarrow{g_{CA}} A$$

The triangle *is* the composite transport:

$$\gamma = g_{CA} \circ g_{BC} \circ g_{AB}$$

Holonomy

- If γ is coherent, the triangle is flat in CAS terms
- If not, its failure to close is geometric information

Key properties

- Angle sums are not fixed
- Curvature is encoded directly in the holonomy class
- Area is not primitive; it is inferred from obstruction

Interpretation

In CAS, **triangles measure curvature directly**. They are probes, not figures.

4. Comparative Summary

Shape	Classical Role	CAS Role
Circle	Fixed-distance locus	Transport-stable radial constraint
Square	Rigid polygon	Coherent symmetry cycle
Triangle	Basic polygon	Minimal holonomy loop

Final Perspective

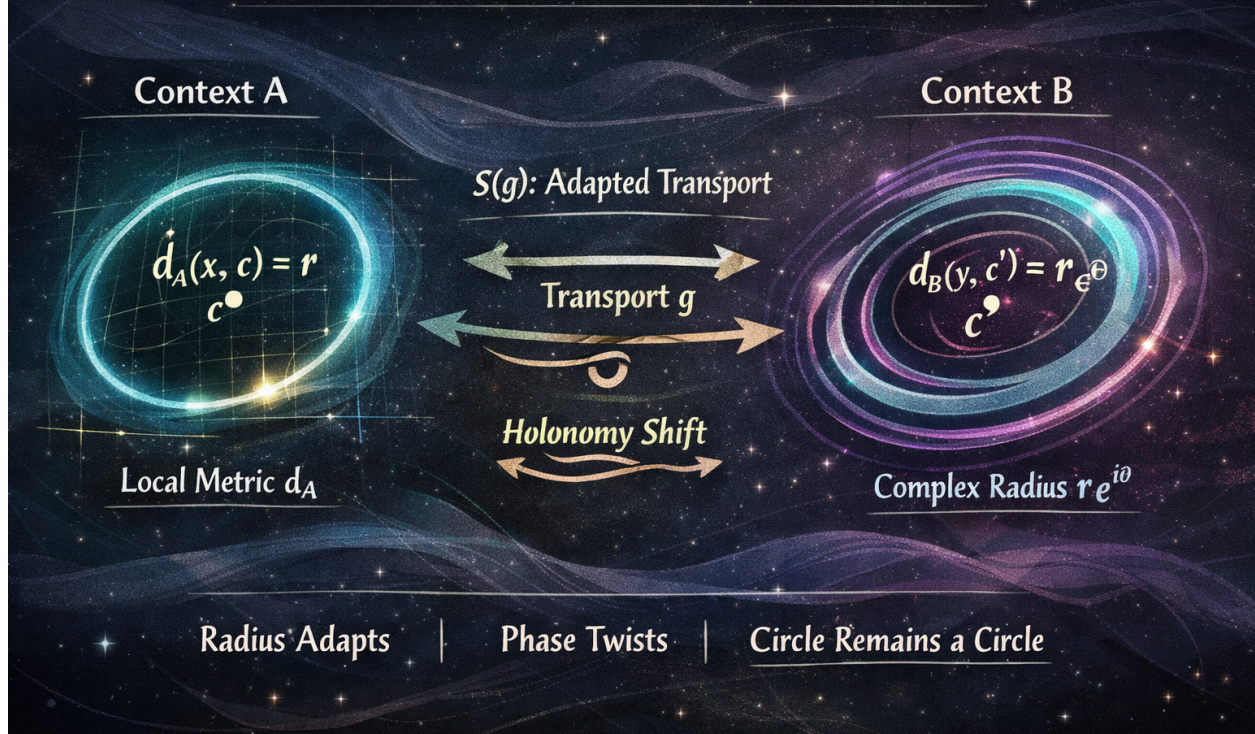
In Complex Adaptive Space:

Shapes are not objects. They are histories of relations surviving motion through context.

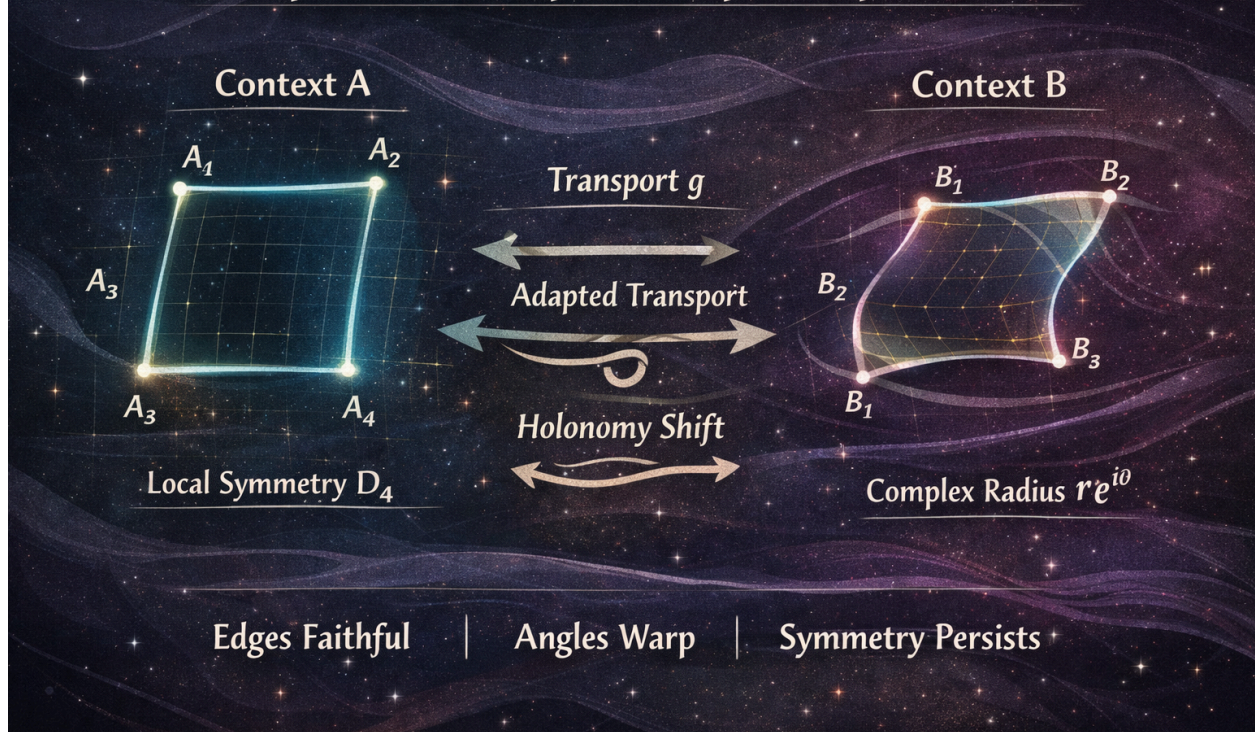
Circles encode invariance, squares encode symmetry, and triangles encode curvature — all through transport and coherence, without requiring fixed coordinates or global constants.

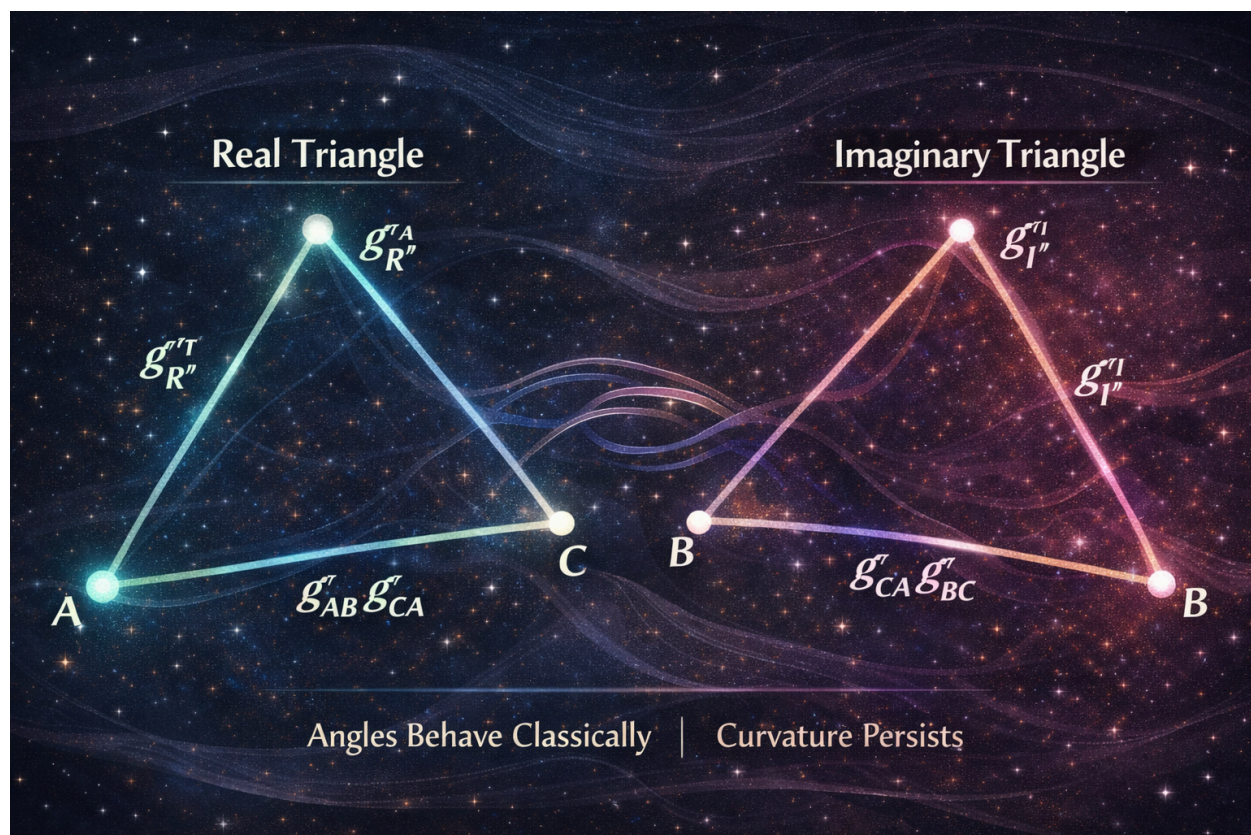
This reframes geometry from *what is drawn* to *what endures*.

Circle in Complex Adaptive Space (CAS)



Square in Complex Adaptive Space (CAS)





Programming in Complex Adaptive Space (CAS) — A Practical Guide

This canvas turns the **minimal axioms** and the **shape probes** of Complex Adaptive Space (CAS) into a programmer's viewpoint: **what to represent**, **what to compute**, and **what to test**.

CAS is **transport-first geometry**: you don't start with global coordinates or a metric. You start with **contexts**, **transports**, **composition**, **coherence/holonomy**, and (often) **branching**.

1) Mental model for coding CAS

The 5 primitives you will encode

1. **Context**: a local "world" where some structure exists (numbers/metrics/operators/etc.).
2. **Transport**: a reversible map that moves structure between contexts (groupoid arrows).
3. **Composition**: path concatenation (transport composition) — order matters.
4. **Coherence**: which loops count as "closing meaningfully" (a predicate or designated subset).
5. **Branching**: when transported values are multi-valued; you must track a chosen sheet/branch.

What a CAS program *does*

- Builds a **graph / groupoid** of contexts and transports.
 - Simulates **paths** and computes **holonomy** (obstruction) as the "payload" of geometry.
 - Treats **shapes** as **tests/probes**:
 - **Circle** = radial relation that persists under transport.
 - **Square** = symmetry cycle that closes coherently.
 - **Triangle** = minimal loop that detects curvature (nontrivial holonomy).
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2) Core data structures

2.1 Context

A context should carry: - **id**: stable identifier - **parameter**: optional complex periodicity parameter $\pi(A) \in \mathbb{C}$ (or your chosen parameter set) - **local structure**: whatever you need locally (norm, metric, operators, basis, etc.) - **sheet / branch state** (if you model branching at the context level)

Recommended representation: - `Context` object with: - `param: complex | tuple` - `sheet: int | label` - `locals: dict` (pluggable local structures)

2.2 Transport (groupoid arrow)

A transport should carry: - **source, target** contexts - an **invertible** mapping (when defined) - a way to **pushforward/pullback** local structure - optional **cost/length** if you add it later

Recommended representation: - `Transport` object with: - `src, dst` - `apply(x, branch_state) -> (value, new_branch_state)` - `inverse()` - metadata: `label`, `domain_constraints`, etc.

2.3 Path

A path is a list of transports: $g_n \circ \dots \circ g_2 \circ g_1$. You will compute: - composed transport action on values
- whether path is a **loop** (ends where it starts) - its **holonomy element / obstruction**

Representation: - `Path = [Transport, ...]`

3) Minimal API (what functions you want)

3.1 Groupoid operations

- `compose(h, g) -> hg` (only if codomain/domain match)
- `identity(A) -> id_A`
- `inverse(g) -> g^{-1}`

3.2 Structure transport

Axiom A suggests each context has some local structure $S(A)$ and transports induce maps $S(g) : S(A) \rightrightarrows S(B)$.

So implement: - `pushforward(g, x, branch) -> (x', branch')` - `pullback(g, y, branch) -> (y', branch')`

3.3 Coherence / holonomy

You need one of these approaches:

Option 1: Predicate coherence - `is_coherent(loop_path, branch_state) -> bool`

Option 2: Designated coherent subgroupoid at each context - `Coh[A]` as a set (or generated set) of coherent loops.

Then define holonomy class: - `holonomy(loop) -> class representative` (e.g., a reduced word, matrix, phase, permutation of sheets)

Practical: start by representing holonomy as a *normal form* of the composed transport around the loop.

3.4 Branching model

Two robust strategies:

Strategy A: Explicit branch state (recommended first) - Carry `branch_state` through every `apply`. - Branch state updates when you cross a cut / change sheet.

Strategy B: Sheeted contexts - Replace each context `A` by `(A, sheet)` so transports become single-valued. - Branching becomes ordinary graph navigation.

4) First implementation choice: discrete vs analytic CAS

Discrete CAS (best for early coding)

- Contexts are nodes in a graph.
- Transports are edges with invertible maps.
- Holonomy is computed by composing edge maps along loops.

Analytic / complex-function CAS (later)

- Context parameter $p \in \mathbb{C}$ (periodicity, exponent, etc.).
- Transports correspond to admissible parameter moves.
- Branching from $z^{1/p}$, $\log z$, etc.

Start discrete; then swap in analytic transports.

5) A small “CAS kernel” you can code

5.1 Interfaces (language-agnostic)

- Context
- Transport
- CAS container holding:
 - contexts
 - transports_by_pair
 - coherence_rule
 - branch_model

5.2 What counts as a loop?

A loop is any path starting and ending at the same context (or same sheeted context).

5.3 What is holonomy computationally?

Compute `G = compose_all(path)`. - If `G` equals identity *on the relevant branch* → trivial holonomy.
- Otherwise, the difference is the holonomy payload.

In code, holonomy can be: - a matrix (linear action) - a complex phase multiplier - a permutation of sheets - a symbolic word in generators

6) Encoding CAS shapes as programs (circles, squares, triangles)

6.1 Circle probe (transport-stable radial constraint)

In each context A , you have a local distance/norm d_A . A circle constraint is: - $d_A(x, c_A) = r_A$

Under transport $g : A \rightarrow B$, you push it forward to: - $d_B(x, c_B) = r_B$ where r_B is induced by transport (possibly branch-dependent).

Program test: pick a point x satisfying the circle constraint in A , transport it, and verify the transported constraint holds in B under the same branch tracking.

Outputs you want: - radius update law $r_B = \text{update\_radius}(g, r_A, \text{branch})$ - whether "circle-ness" is preserved up to coherence

6.2 Square probe (coherent symmetry cycle)

A square is a **4-step transport cycle**: $A_1 \xrightarrow{g_1} A_2 \xrightarrow{g_2} A_3 \xrightarrow{g_3} A_4 \xrightarrow{g_4} A_1$

Program test: - Compose $g_4 \circ g_3 \circ g_2 \circ g_1$ and check if it is coherent. - Track what changes after one circuit: - rotation / mirror (as an action) - sheet change - phase change

A CAS square is the *equivalence class* of such cycles under coherence.

6.3 Triangle probe (minimal holonomy loop)

Triangle is the smallest nontrivial loop: $\gamma = g_{CA} \circ g_{BC} \circ g_{AB}$.

Program test: - Compute holonomy of γ . - If coherent $\rightarrow$ "flat" (in CAS terms). - If not $\rightarrow$ obstruction is your curvature signal.

This is your main diagnostic for "curvature in context-space."

7) A concrete toy model you can build quickly

Here's a discrete model that already exhibits holonomy and branching:

Contexts

Take contexts $A(p, k)$ where: - p is a complex parameter (or just a real scalar to start) - k is a sheet index (integer mod N)

Transport generators

Define two generator transports: 1. t : increments parameter $p \rightarrow p + \Delta$ (move in parameter space)
2. b : sheet shift $k \rightarrow k + 1$ (cross a branch cut)

Make them invertible: - t^{-1} decrements - b^{-1} shifts back

Holonomy

Define a loop that returns in p but not in k : - $t \circ b \circ t^{-1} \circ b^{-1}$ This can yield a nontrivial sheet action even though the parameter returns.

This is a minimal “branch-holonomy” demonstration.

8) Testing & invariants (how to know your CAS code is right)

Groupoid laws (unit tests)

- $\text{compose}(\text{id}_B, g) == g$ and $\text{compose}(g, \text{id}_A) == g$
- $\text{compose}(g, \text{inverse}(g)) == \text{id}_A$ and $\text{compose}(\text{inverse}(g), g) == \text{id}_B$
- associativity where defined

Coherence consistency

If your axioms include closure: - coherent loops should be closed under composition - conjugation should preserve coherence: $g \circ \gamma \circ g^{-1}$ coherent if γ is

Branch discipline

- Along a chosen branch, values should be locally single-valued.
 - Changing branch must be explicit in state or in sheeted contexts.
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9) Roadmap: from toy code to your CAS papers

Phase 1: Discrete CAS engine

- Implement contexts/transport/path
- Support branch state or sheeted contexts
- Compute holonomy classes

Phase 2: “Shapes as probes” library

- Circle probe (radial constraint tester)
- Square probe (cycle coherence tester)
- Triangle probe (minimal holonomy detector)

Phase 3: Analytic CAS transports

- Complex periodicity parameter p
- Transport of quantities like $c^{1/p}$ with branch tracking
- Branch cuts as first-class geometry

Phase 4: Invariants

- define a cost/length on transports
- define conserved observables up to coherence

- catalog holonomy types (phase, permutation, matrix action)

10) Worked example (matching the circles / squares / triangles document)

This is a complete, explicit toy CAS model that exhibits: - context-dependent structure, - branching as first-class data, - coherence as a designated subset of loops, - holonomy computed on loops.

It is intentionally minimal, but it matches the meanings in your shapes document.

10.1 Contexts and local structure

Contexts - Objects: $Ob = \{ A_p : p \in P \subset C \setminus \{0\} \}$. - Parameter map: $\pi(A_p) = p$.

Local structure at each context - Point space: $S(A_p) = C$ (points are complex numbers). - Branch index: n in Z (sheet counter). - A distinguished isotropic “growth” observable (multi-valued unless you track n). We choose $c = -1$ so phases are visible:

$$f(p; n) := \exp(i * (\pi + 2\pi n) / p)$$

This is a concrete version of “ $c^{1/p}$ with branch management”.

10.2 Explicit transport rule (with branching)

A transport $g: A_p \rightarrow A_q$ acts on a transported pair (z, n) as:

$$(T) \text{ Transport rule - } z' = z - n' = n + w(g)$$

where $w(g)$ in Z is the winding / cut-crossing count attached to that transport.

In the target context, the carried observable is evaluated as $f(q; n')$.

Composition - If $h: A_q \rightarrow A_r$ then $w(h \circ g) = w(h) + w(g)$, and branch states add accordingly.

Identity / inverse - $w(id_A) = 0$ - $w(g^{-1}) = -w(g)$

10.3 Coherence and holonomy (explicit)

For any loop γ (a path starting and ending at the same context), define net winding:

$$W(\gamma) := \sum \text{ over edges } g \text{ in } \gamma \text{ of } w(g).$$

Coherence rule (simple and explicit) - γ is coherent at A_p iff $W(\gamma) = 0$.

Holonomy payload - Holonomy is detected by $W(\gamma)$ (or equivalently by the induced change of branch/phase on f). - “Failure to return” is geometric information.

10.4 Circles in this model (transport-invariant radial relations)

The shapes document says: - each context has its own distance/norm, - circle constraints push forward under transport, - the radius may become complex- or branch-dependent.

We implement that directly.

Local CAS-distance - $d_{\{p,n\}}(z, c) := |z - c| / |f(p; n)|$

Circle in A_p Pick center c in C and local radius $R > 0$. Define the circle-locus: - $C_{\{p,n\}}(c, R) := \{ z \in C : d_{\{p,n\}}(z, c) = R \}$

Equivalently (ambient units): - $|z - c| = R * |f(p; n)|$

Pushforward under $g: A_p \rightarrow A_q$ Under (T) , z and c are unchanged but $n \rightarrow n' = n + w(g)$. So the pushed-forward constraint is: - $d_{\{q,n'\}}(z, c) = R$ which is - $|z - c| / |f(q; n')| = R$.

So circle-ness persists, while the “ambient radius” adapts: - $r_{\{p,n\}} = R * |f(p; n)|$ maps to $r_{\{q,n'\}} = R * |f(q; n')|$.

If you allow a complex-valued radius (often natural in CAS): - $\rho_{\{p,n\}} := R * f(p; n)$ maps to $\rho_{\{q,n'\}} := R * f(q; n')$.

Concrete numeric circle demo Take $p = 2$, $q = 3$, start branch $n = 0$, choose $R = 2$, center $c = -1$. - $f(2;0) = \exp(i\pi/2) = i$ so $\rho_{\{2,0\}} = 2i$. - If $w(g) = 0$ then $n' = 0$ and $f(3;0) = \exp(i\pi/3)$, so $\rho_{\{3,0\}} = 2\exp(i\pi/3)$. - If you then take a cut-crossing transport $b: A_3 \rightarrow A_3$ with $w(b) = 1$, then $n \rightarrow n+1$ and $f(3;1) = \exp(i(\pi+2\pi)/3) = \exp(i\pi) = -1$, so $\rho_{\{3,1\}} = -2$.

Same radial relation; different branch/phase.

10.5 Triangles in this model (minimal holonomy loops)

A CAS triangle is the smallest nontrivial closed transport loop.

Build a triangle loop Let the loop be: - $A_2 \xrightarrow{g} A_3 \xrightarrow{b} A_3 \xrightarrow{h} A_2$ with - $w(g) = 0$ - $w(b) = 1$ - $w(h) = 0$

Holonomy - $W(\gamma) = 0 + 1 + 0 = 1 \neq 0$. So γ is not coherent and its holonomy is nontrivial.

What fails to return? Points return (toy choice), but the branch state does not: - start: $f(2;0) = \exp(i\pi/2) = i$ - after loop: branch $n \rightarrow n+1$, so $f(2;1) = \exp(i(\pi+2\pi)/2) = \exp(i*3\pi/2) = -i$

So a transported complex-radius $\rho = R*f$ flips sign: $2i \rightarrow -2i$.

Interpretation: the triangle detected curvature directly as holonomy/obstruction.

10.6 Squares in this model (coherent symmetry cycles)

A CAS square is a closed 4-step transport cycle, considered up to coherence.

A branch-shifted (non-coherent) square Take the 4-step loop: $- A_2 \xrightarrow{g} A_3 \xrightarrow{b} A_3 \xrightarrow{g^{-1}} A_2 \xrightarrow{id} A_2$ with $- w(g) = 0 - w(b) = 1 - w(g^{-1}) = 0 - w(id) = 0$

Net winding: $- W(\text{square}) = 1 \neq 0$. So after one circuit, the square returns branch-shifted (holonomy present). In different contexts it may appear skewed/curved; what matters is the loop's transport class.

A coherent square Use opposite cut-crossings so they cancel: $- A_2 \xrightarrow{g} A_3 \xrightarrow{b} A_3 \xrightarrow{b^{-1}} A_2 \xrightarrow{g^{-1}} A_2$ Now $- W(\text{square}) = 0 + 1 - 1 + 0 = 0$. So the 4-cycle is coherent: it closes "meaningfully" in CAS terms.

11) How this scales to richer CAS models

This worked example is a minimal instantiation of:
- groupoid transports (invertible when defined),
- adaptivity (structure comparisons mediated by transports),
- coherence/holonomy (loops need not be trivial),
- branching (multi-valuedness treated structurally),
- optional isotropic structure $f(A) = c^{1/p}$ with branch tracking.

Next upgrade if you want the figures to visibly deform in the ambient plane:
- let transports act nontrivially on points, e.g. $z' = \alpha(p,q,n)z + \beta(p,q,n)$ or a nonlinear map, while keeping the same branch-tracked transport on f . Then circles can look skewed while remaining CAS-circles by the transported constraint.

12) What I can write next (pick any direction)

- A reference implementation skeleton (Python/JS) that computes W , checks coherence, and reports holonomy on loops.
- A version where transports are analytic (log/power) and sheeted contexts eliminate multi-valuedness.
- A small library of probes: `circle_probe`, `square_probe`, `triangle_probe` with test outputs.

Glossary of Terms — Complex Adaptive Space (CAS)

This glossary consolidates and standardizes terminology used across the following CAS documents: - *Complex Adaptive Space — Preface and Reader's Guide* - *Complex Adaptive Space — Minimal Axioms - Circles, Squares, and Triangles in Complex Adaptive Space* - *Programming in Complex Adaptive Space (CAS) — A Practical Guide*

The goal is conceptual clarity rather than encyclopedic completeness. Definitions are intentionally **operational**: each term is defined by how it functions inside CAS.

A

Adaptivity (Complex Adaptivity)

The principle that **laws, structures, or observables are not globally fixed**, but vary with context. Comparisons between contexts are only meaningful via transport. There is no absolute identification of objects across contexts.

Axiom A (Adaptivity Axiom)

The axiom stating that each context supports its own local structure, and that transports induce pushforward/pullback maps between these structures. All cross-context comparison is mediated by transport.

B

Branch

A locally chosen determination of a multi-valued quantity. Along a chosen branch, transported quantities are single-valued, but globally multiple branches may exist.

Branch Cut

A structural obstruction where continuation forces a change of branch. In CAS, branch cuts are **geometric data**, not artifacts to be removed.

Branching (Axiom B)

The axiom asserting that transported quantities may be multi-valued, but must be locally single-valued along any admissible path or chosen sheet. Branching is intrinsic, not pathological.

C

CAS (Complex Adaptive Space)

A transport-first geometric framework in which geometry is defined by how structure changes across contexts, rather than by fixed coordinates, metrics, or global constants.

Circle (CAS Circle)

Not a fixed-distance locus, but a **transport-stable radial relation**. A CAS circle is a family of local constraints that remain meaningful under transport, possibly with changing radius, phase, or branch.

Coherence

A rule or predicate specifying which closed transport loops are considered to “close meaningfully.” Coherence distinguishes genuine geometric closure from obstruction.

Coherent Loop

A closed transport path designated as coherent under the coherence rule. Coherent loops represent symmetry or invariance rather than curvature.

Context

A local “world” in which some structure exists. Contexts are the objects of the CAS groupoid. Each context may have its own parameters, laws, metrics, or observables.

Curvature (in CAS)

Not a tensor or scalar field, but **holonomy**: the failure of transported structure to return after traversing a closed loop in context-space.

G

Groupoid

An algebraic structure consisting of objects (contexts) and invertible arrows (transports) defined only when domains and codomains match. CAS contexts and transports form a groupoid.

Groupoid Axiom (Axiom G)

The axiom asserting that transports are locally invertible and composable where defined, with identities and inverses. This is the minimal structural backbone of CAS.

H

Holonomy

The equivalence class of a closed transport loop modulo coherent loops. Holonomy measures obstruction, curvature, or failure to return. In CAS, holonomy *is* curvature.

Holonomy Class

The residue of a loop after factoring out coherent closure. Can be represented as a phase shift, branch permutation, matrix action, or other invariant.

I

Isotropic Scaling Structure (Optional)

An additional structure often used in CAS, where each context carries a distinguished invariant (e.g. a constant c) and a context-dependent growth law (such as $c^{1/p}$), with branch management.

L

Loop

A transport path that starts and ends at the same context (or same sheeted context). Loops are the primary carriers of geometric information in CAS.

P

Path

A finite composition of transports. Paths encode history and order dependence; reversing or reordering a path generally changes its meaning.

Parameter (Context Parameter)

A value (often complex) associated to a context that influences its local structure. Parameters commonly encode periodicity, exponentiation, or phase behavior.

Pushforward / Pullback

The induced action of a transport on local structure. Pushforward moves structure forward along a transport; pullback moves it backward.

S

Sheet

A refinement of a context used to make multi-valued transport single-valued. Sheeted contexts are an alternative formalization of branching.

Shape (in CAS)

Not a static object, but a **persistent relational pattern** under transport. Shapes function as probes that reveal properties of the space.

Square (CAS Square)

A **coherent four-step transport cycle** representing symmetry. A CAS square is the equivalence class of such cycles under coherence, not a rigid polygon.

T

Transport

An invertible (when defined) mapping that moves structure between contexts. Transports are the fundamental dynamical elements of CAS geometry.

Transport-First Geometry

A philosophical stance: geometry is defined by motion, comparison, and transformation, not by static background space.

Triangle (CAS Triangle)

The smallest nontrivial closed transport loop. A CAS triangle is a curvature detector: if it fails to close coherently, it reveals holonomy directly.

W

Winding (Net Winding)

An integer or discrete measure of how many times a path crosses branch cuts or cycles through sheets. Often used as a concrete holonomy invariant in examples.

Z

Zero Structure (Implicit)

CAS does not privilege global constants such as zero, length, or angle. Any such notion must be defined contextually or via transport-invariant relations.

Final Note

This glossary is intentionally minimal and transport-centric. Terms are defined by **what they do**, not by analogy to classical geometry. If a definition cannot be tested by transport, composition, or coherence, it is not fundamental in CAS.

Epilogue: On Movement, Space, and Meaning

This work has deliberately begun with movement rather than space. That choice is not merely technical; it reflects a deeper philosophical stance about what geometry is, and where its meaning comes from.

Classical geometry presupposes space as a fixed container within which objects exist and move. Motion is then treated as secondary: a change of position measured against an already-given background. Complex Adaptive Space (CAS) inverts this order. It does not assume a global space in which motion occurs. Instead, it asks how notions of sameness, persistence, and structure arise when only contexts and transformations are given. Space, in CAS, is not primary—it is induced.

This inversion places CAS at a subtle but important metaphysical boundary. It moves away from naïve physical realism, in which geometry is understood as an objective, static property of the world independent of relational history. In CAS, geometry is not what exists prior to interaction; it is what survives interaction. Curvature is not deviation from an ideal flatness but the memory of a journey. Shape is not an object but a pattern of coherence.

At the same time, CAS does not reduce geometry to subjective perception or mental construction. Nothing in the framework requires an observer, a mind, or an act of measurement. Transports, coherence, and branching are structural, not psychological. In this sense, CAS does not assert idealism. Rather, it aligns most naturally with a relational or structural realism: a view in which relations and transformations are ontologically primary, and objects or spaces are secondary stabilizations of those relations.

It is nonetheless no accident that CAS resonates strongly with features of conscious experience. Consciousness does not encounter the world as a fixed coordinate grid. Meaning shifts with context. Identity persists through transformation rather than stasis. History matters; paths cannot always be undone without residue. Ambiguity and multivaluedness are not errors but facts of lived structure. CAS mirrors these features because it is built from the same primitive question: what does it mean for something to remain meaningful when the conditions of meaning themselves can change?

In this sense, CAS sits closer to process-based and relational philosophies of nature than to substance-based ones. It treats becoming as prior to being, transport as prior to location, and coherence as prior to invariance. Space appears not as a stage, but as an emergent record of what has endured.

The aim of this work has not been to replace existing geometric or physical theories, but to offer a minimal language capable of expressing phenomena that those theories often treat as exceptional: path dependence, contextual laws, branching structure, and nontrivial return. By beginning with movement rather than space, CAS reframes geometry as an inquiry into persistence under transformation—a stance that is mathematically precise, physically compatible, and philosophically open.

If classical geometry asks what can be drawn, Complex Adaptive Space asks what can be carried, returned, and recognized again. In that shift lies both its mathematical utility and its metaphysical significance.